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Root-Mean-Square Miss Distance of Proportional Navigation Missile Against Sinusoidal Target

Ernest J. Ohlmeyer*
U.S. Naval Surface Warfare Center, Dahlgren, Virginia 22448-5000

The performance of a proportional navigation homing missile against a target performing a sinusoidal weave maneuver is evaluated. The missile's effectiveness is measured in terms of the root-mean-square miss distance over a set of engagements in which the initial phase of the target weave is uniformly distributed. Closed form solutions for the root-mean-square miss are derived for the case where the missile guidance system is modeled by a first-order lag and the lateral acceleration is unlimited. The analysis is then extended to include the effects of acceleration saturation and higher order missile dynamics. Comparisons are made between a first-order and a fifth-order guidance system, and the root-mean-square miss is determined numerically as a function of the interceptor's effective navigation gain, time constant and acceleration limit, and the target's weave amplitude and frequency.

Introduction

FUTURE homing interceptor missiles will face new and unique challenges as the sophistication of the threat spectrum increases. Engagements against air targets can occur at both very low and very high altitudes, with the threats accidentally or intentionally performing weaving or spiraling maneuvers during their midcourse and terminal phases.[1–4] The lateral displacement, acceleration capability, and weave frequency of the target maneuver can greatly enhance the threat's ability to survive a counterattack.

To counter this, the defensive missile must have sufficient lateral acceleration, guidance system time constant, and terminal homing time to achieve a high probability of intercept. Whereas the performance of the interceptor will be influenced by many scenario-dependent factors,[5–8] a major consideration will be the fundamental response of the proportional navigation (PN) guidance system to the postulated target weave motion.[9,10]

In the general case, the target dynamics may involve arbitrary periodic motion in three dimensions. A useful starting point for analysis, however, is the response of the PN homing system to a single plane sinusoidal maneuver of constant amplitude and frequency. The phase angle of target weave, which is associated with initial conditions at the start of the missile's terminal guidance, can be treated as a random variable, uniformly distributed between 0 and 2π over a set of engagements. The missile's dynamics are approximated by a simple first-order transfer function, and unlimited lateral acceleration capability is assumed. The miss distance can then be parameterized in terms of the effective PN navigation gain N, the missile time constant τ, and the amplitude A_T and frequency ω of the target weave.

This paper focuses on root-mean-square (rms) miss distance as a recommended measure of effectiveness in analyzing missile performance against weaving targets. This measure allows uncertainties in target phase characteristics to be accounted for in the terminal performance results. The weaving target problem was first addressed by Chadwick,[9] who determined analytical expressions for the rms miss distance of the single lag PN missile for values of $N = 2$ and 3. Zarchan[10] employed adjoint theory and transfer function techniques to determine formulas for the peak miss distance against a weaving target for values of N between 3 and 6. The present paper derives general closed-form expressions for the rms miss distance against a sinusoidal target. New results are obtained for arbitrary N that agree with the particular results of Ref. 9, but with a less complicated solution method. In addition, two alternative methods for normalizing the miss distance data are discussed.

Extension of previous results on acceleration saturation is also addressed. Reference 10 considered the effects of acceleration limiting on the peak miss distance for a fifth-order guidance system. The present work extends the analysis to include the effects of acceleration saturation on the rms miss distance for both a first-order and a fifth-order guidance system using Monte Carlo simulation techniques. New results are provided that compare the two guidance systems in terms of achievable performance and sensitivity to key parameters.

Miss Distance Solutions via Adjoint Techniques

Consider the simplified diagram of the linearized missile homing loop shown in Fig. 1, where t_{go} is time to go, a_m and a_T are missile and target accelerations, y is lateral separation, V_c is closing velocity, and λ is line-of-sight angle. In this figure, the dynamic responses of the airframe/autopilot, guidance noise filters and seeker track loop have been combined into a simple first-order transfer function representation with time constant τ. The PN guidance law is implemented as $a_c = N V_c \,\mathrm{d}\lambda/\mathrm{d}t$.

The response of the homing loop to the target acceleration and the resulting terminal miss distance may be analyzed through the use of adjoint and Laplace transform techniques.[11–14] Using standard adjoint methods, Fig. 1 may be rearranged into the form shown in Fig. 2. Here the elements of the block diagram include a time-dependent block $1/t^*$ (where t^* is adjoint time), a grouping of elements denoted as $W(s)$, and a function f representing a time-dependent scaling of a unit step acceleration.

From Fig. 2, the miss distance at final time t_f may be written

$$M(t_f) = \int_0^{t_f} f(t_f - t^*) a_T(t^*)\,\mathrm{d}t^* \tag{1}$$

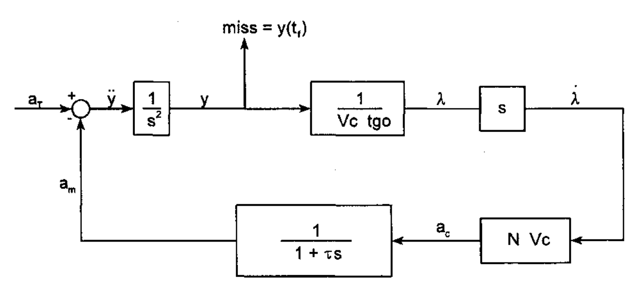

Fig. 1 First-order homing loop diagram.

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*Group Leader, Guidance and Control, Missile Systems Division. Senior Member AIAA.

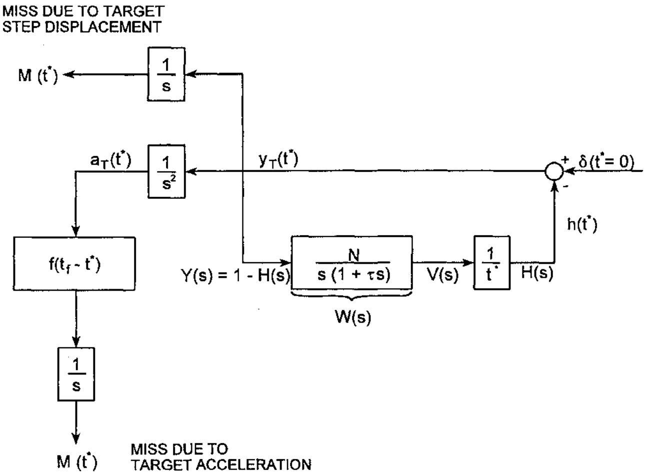

Fig. 2 Adjoint diagram for generating miss distance.

Equation (1) is in the form of a convolution integral. From linear systems theory, this may be represented in the frequency domain as

$$M(s) = F(s)A_T(s) = \frac{F(s)Y(s)}{s^2} \tag{2}$$

where $F(s)$ and $Y(s)$ are Laplace transforms of the functions $f(t)$ and $y_T(t)$. Zarchan[11] has shown that for a first-order guidance system, $Y(s)$ satisfies the relation

$$Y(s) = \exp\left[\int W(s)\,\mathrm{d}s\right] = \left(\frac{s}{s+1/\tau}\right)^N \tag{3}$$

In this example, obtaining $Y(s)$ in closed form relied on the fact that $\int W(s)\,\mathrm{d}s$ was logarithmic. For guidance systems higher than first order, this will not be true, and $Y(s)$ will not be expressable as a ratio of polynomials in s. Hence, simple analytic solutions will not exist.

Combining Eqs. (2) and (3) gives the result[11]

$$G(s) = \frac{M(s)}{F(s)} = \frac{1}{s^2}\left(\frac{s}{s+1/\tau}\right)^N \tag{4}$$

The transfer function $G(s)$ relates the final miss distance to the input target acceleration for the first-order guidance system. The miss at time t_f could, in principle, be obtained by taking the inverse Laplace transform of $M(s)$ for any specified target acceleration with transform $F(s)$. An example would be the $F(s)$ corresponding to $f(t) = A_T \sin \omega t$.

However, it is not necessary to actually invert the Laplace transform to obtain the maximum miss or the rms miss in steady state. Here, steady state refers to the condition where all transients have disappeared and the relation $\tau/t_f \ll 1$ is satisfied. The frequency response properties of $G(s)$, i.e., the amplitude ratio and phase lag, can be employed to derive the steady-state miss distance. In Ref. 10, the author uses the magnitude of $G(j\omega)$ at selected N to characterize the peak miss distance response to a weave input. In the following sections, the rms miss distance is derived using frequency domain concepts, combined with additional statistical analysis.

Both the magnitude and phase of $G(j\omega)$ can be written out explicitly for arbitrary N. The magnitude of $G(j\omega)$ is given by

$$|G(j\omega)| = \left|\frac{(j\omega)^{N-2}}{(j\omega + 1/\tau)^N}\right| \qquad N \geq 2$$

To evaluate this, one may form the ratio of products

$$|G(j\omega)| = \left(\prod_{i=1}^{N-2} |j\omega|\right) \bigg/ \left(\prod_{i=1}^{N} |j\omega + 1/\tau|\right) = \frac{\omega^{N-2}}{[\omega^2 + (1/\tau)^2]^{N/2}}$$

Table 1 Magnitude and phase of $G(j\omega)$ for various N

N	Magnitude, K	Phase, ϕ
2	$\frac{\tau^2}{1+(\omega\tau)^2}$	$-2\tan^{-1}(\omega\tau)$
3	$\frac{\omega\tau^3}{[1+(\omega\tau)^2]^{3/2}}$	$\frac{\pi}{2} - 3\tan^{-1}(\omega\tau)$
4	$\frac{\omega^2\tau^4}{[1+(\omega\tau)^2]^2}$	$\pi - 4\tan^{-1}(\omega\tau)$
5	$\frac{\omega^3\tau^5}{[1+(\omega\tau)^2]^{5/2}}$	$\frac{3\pi}{2} - 5\tan^{-1}(\omega\tau)$
6	$\frac{\omega^4\tau^6}{[1+(\omega\tau)^2]^3}$	$2\pi - 6\tan^{-1}(\omega\tau)$

which yields

$$K = |G(j\omega)| = \frac{\omega^{N-2}\tau^N}{[1+(\omega\tau)^2]^{N/2}} \tag{5}$$

The phase angle of $G(j\omega)$ may be written as

$$\angle G(j\omega) = \sum_{i=1}^{N-2} \angle j\omega - \sum_{i=1}^{N} \angle(j\omega + 1/\tau)$$

which results in

$$\phi = \angle G(j\omega) = (N-2)(\pi/2) - N\tan^{-1}(\omega\tau) \tag{6}$$

Table 1 shows the resulting expressions for K and ϕ for values of N between 2 and 6.

As noted in Ref. 10, the transfer function $G(s)$ may be physically interpreted as follows: If the linear homing system is driven by a sinusoidal input of frequency ω, then the output final miss distance (after transients have disappeared) is a sinusoidal function of ωt_f with an amplitude that is K times the input amplitude and with a phase lag ϕ. That is, the steady-state miss for an input of $A_T \sin \omega t$ is given by

$$M(t_f) = K A_T \sin(\omega t_f + \phi) \tag{7}$$

Maximum and RMS Miss Distance Against Weaving Target

The result of Eq. (7) can be used to predict the worst-case magnitude of the steady-state miss distance over all t_f, which is just

$$M_{\max} = K A_T \tag{8}$$

Note from Table 1 that two nondimensional factors for plotting miss distance results are suggested, namely, that Eq. (8) may be put in either of the alternate forms

$$\frac{M_{\max}}{A_T\tau^2} = f_1(\omega\tau) = \frac{(\omega\tau)^{N-2}}{[1+(\omega\tau)^2]^{N/2}}$$

and

$$\frac{M_{\max}\omega^2}{A_T} = f_2(\omega\tau) = \frac{(\omega\tau)^N}{[1+(\omega\tau)^2]^{N/2}}$$

As $\omega\tau$ becomes very large, these expressions asymptotically approach the functions $1/(\omega\tau)^2$ and 1, respectively. This implies that the peak miss will approach zero as the target maneuver frequency becomes much larger than the guidance system bandwidth $(1/\tau)$.

An alternate measure of effectiveness for evaluating terminal homing performance against weaving targets is the rms miss distance. Consider the sketch in Fig. 3a, where the linear system $G(s)$ is driven by an input

$$a_T(t^*) = A_T \sin(\omega t^* + \theta) \tag{9}$$

Let the phase angle θ of the sinusoidal target acceleration be a random variable between 0 and 2π with a uniform probability density as shown in Fig. 3b. Then the rms value of the steady-state miss

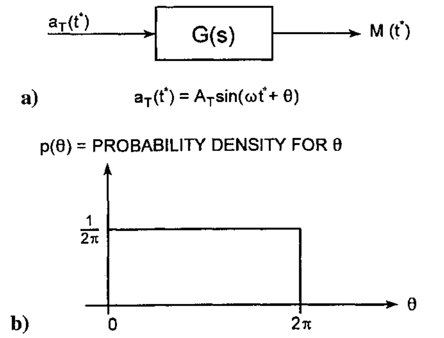

Fig. 3 Sinusoidal target acceleration with uniformly distributed random phase angle.

distance over an ensemble of engagements may be determined in the following manner.

Expanding Eq. (9) yields

$$a_T = A_T \cos\theta \sin\omega t^* + A_T \sin\theta \cos\omega t^*$$

Let the transfer function $G(j\omega)$ be represented in terms of an amplification factor K and a phase lag ϕ, which are both functions of ω. The steady-state miss distance then becomes

$$M(t^*) = A_T K \cos\theta \sin(\omega t^* + \phi) + A_T K \sin\theta \cos(\omega t^* + \phi)$$

Now take an ensemble average (at fixed t^*) over the random variable θ:

$$\begin{aligned} E(M^2) &= A_T^2 K^2 E(\cos^2\theta)\sin^2(\omega t^* + \phi) \\ &\quad + A_T^2 K^2 E(\sin^2\theta)\cos^2(\omega t^* + \phi) \\ &\quad + 2A_T^2 K^2 E(\cos\theta\sin\theta)\sin(\omega t^* + \phi)\cos(\omega t^* + \phi) \end{aligned} \tag{10}$$

where E denotes the expectation operator.

Using the uniform probability density $p(\theta)$ in Fig. 3b, it can be easily shown that

$$E(\cos^2\theta) = \int_0^{2\pi} \cos^2\theta p(\theta)\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta\,d\theta = \frac{1}{2}$$

Similarly, one obtains

$$E(\sin^2\theta) = \tfrac{1}{2} \qquad E(\cos\theta\sin\theta) = 0$$

Thus, Eq. (10) reduces to

$$E(M^2) = \tfrac{1}{2}A_T^2 K^2[\sin^2(\omega t^* + \phi) + \cos^2(\omega t^* + \phi)] = \tfrac{1}{2}A_T^2 K^2$$

The rms miss is given by the square root of this expression or

$$M_{\text{rms}} = A_T K/\sqrt{2} \tag{11}$$

Thus for the assumed statistical distribution, and for unlimited missile acceleration, the rms miss is related to the peak miss by

$$M_{\max}/M_{\text{rms}} = \sqrt{2} = 1.414$$

which is also the ratio of the maximum amplitude of a sine wave to its rms value. (For the case of saturated missile acceleration, the relation between peak miss and rms miss must be determined numerically, especially for smaller values of $\omega\tau$.) Equation (11) indicates that the rms miss is independent of time. That is, if at any time t^* (after transients have gone) one takes a snapshot of the output $M(t^*)$ and does an average over all possible values of θ, then the same answer is obtained regardless of t^*. In statistical language, this describes an ergodic random process.

The earlier derived expression for K may be substituted into Eq. (11) to yield

$$M_{\text{rms}} = \frac{A_T}{\sqrt{2}}\frac{\omega^{N-2}\tau^N}{[1 + (\omega\tau)^2]^{N/2}} \tag{12}$$

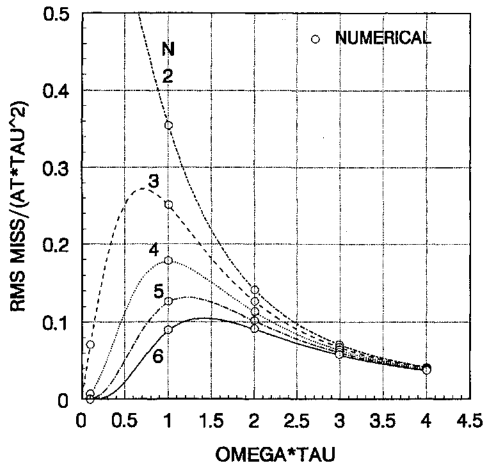

Fig. 4 Comparison of rms miss distance results with numerical computations (first normalization).

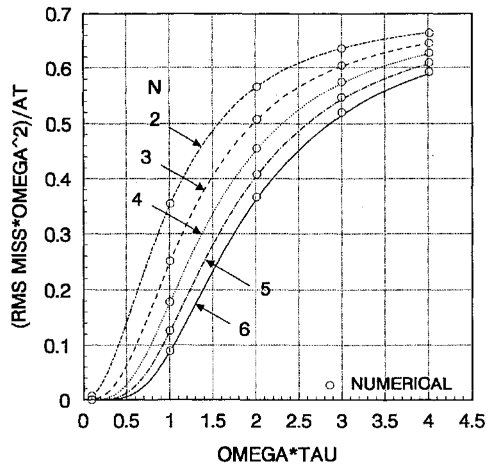

Fig. 5 Comparison of rms miss distance results with numerical computation (second normalization).

As before, this can be put into either of two nondimensional forms:

$$\frac{M_{\text{rms}}}{A_T\tau^2} = \frac{(\omega\tau)^{N-2}}{\sqrt{2}[1 + (\omega\tau)^2]^{N/2}} \tag{13}$$

$$\frac{M_{\text{rms}}\omega^2}{A_T} = \frac{(\omega\tau)^N}{\sqrt{2}[1 + (\omega\tau)^2]^{N/2}} \tag{14}$$

The expressions in Eqs. (13) and (14) for the rms miss were numerically verified by using forward time digital simulation of the single lag PN homing system. The equations of motion were solved using a fourth-order Runge–Kutta integration scheme with a time step of 0.0005 s. Over a set of 100 Monte Carlo runs, the initial phase angle of the target weave was randomly varied between 0 and 2π, using a uniform distribution. The numerical computations were performed over a range of parameter values for τ, ω, A_T, N, and t_f. Figures 4 and 5 show the very good agreement obtained between the numerical results and the analytical formulas for values of N between 2 and 6.

Acceleration Limiting

The preceding results were generated under the assumption that the missile had unlimited lateral acceleration capability. In reality, if the target maneuver becomes severe enough, the missile acceleration will saturate. The performance of the single lag PN homing system was evaluated by limiting the commanded missile acceleration over a range of values and determining the effect on the rms miss distance. The limits were applied such that $|a_c| \le A_{\text{lim}}$. The miss distance results were obtained from numerical Monte Carlo simulations since closed-form solutions do not exist in the case of limited acceleration.

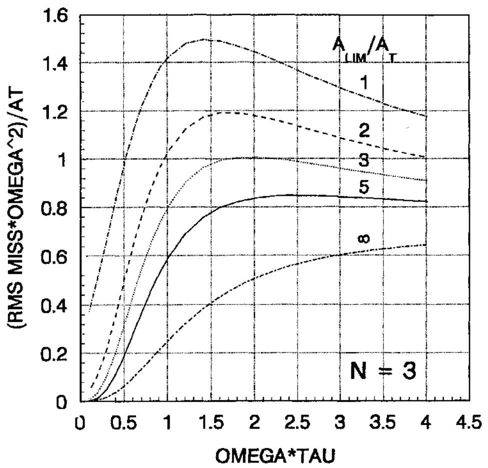

Fig. 6 RMS miss distance of first-order guidance system with acceleration limits.

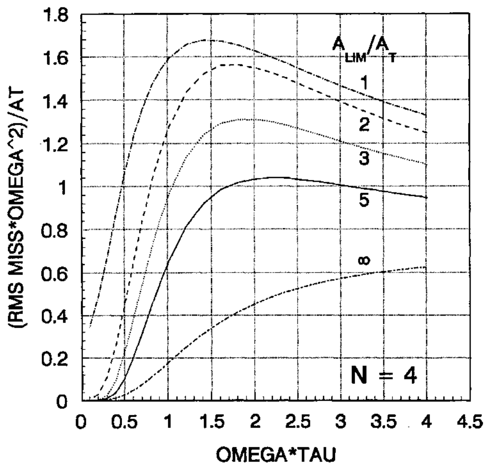

Fig. 7 RMS miss distance of first-order guidance system with acceleration limits.

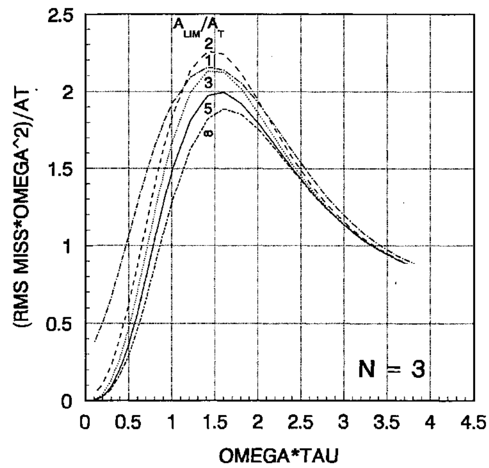

Fig. 8 RMS miss distance of fifth-order guidance system with acceleration limits.

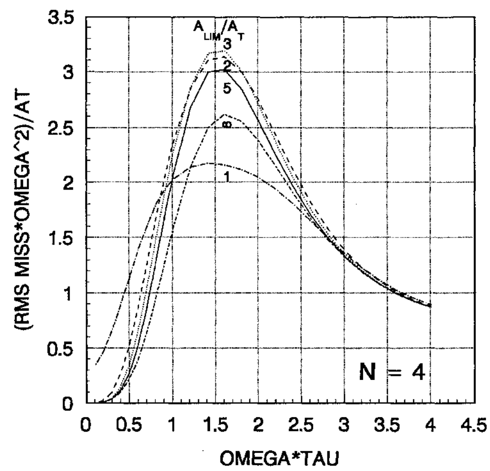

Fig. 9 RMS miss distance of fifth-order guidance system with acceleration limits.

The effects of acceleration limits on the first-order guidance system are shown in Figs. 6 and 7 for $N = 3$ and 4 and for ratios of $A_{\lim}/A_T$ of 1, 2, 3, 5, and ∞. The same two miss distance normalization factors that were used for the unlimited acceleration case were used to evaluate the saturated guidance system. To be concise, however, only results for the $M_{\rm rms}\,\omega^2/A_T$ factor are presented.

Because the saturated homing system is nonlinear, checks were performed to ensure that the normalization results remained valid over the expected range of target and missile parameters. Simulation results for three levels of target acceleration (5, 10, and 15 g) were shown to yield the same normalized miss distance for a range of $\omega\tau$ values.

The results in Figs. 6 and 7 indicate that acceleration limiting has a pronounced effect on the rms miss distance. This effect is very noticeable at smaller $\omega\tau$ for the factor $M_{\rm rms}/(A_T\tau^2)$ but is clearly seen over the full $\omega\tau$ range using the factor $M_{\rm rms}\omega^2/A_T$. For navigation gains of 3 and 4, the trend is for normalized miss distance to increase as the ratio of limited acceleration to target acceleration decreases from infinity (no limit) to one. For $N = 5$ (not shown) a reversal in the miss distance trend occurs. For $\omega\tau$ greater than about 1, the miss distances peak at $A_{\lim}/A_T = 2$ and then decrease somewhat for $A_{\lim}/A_T = 1$.

Higher-Order Guidance Systems

A similar kind of analysis can be used to determine miss distance performance for a missile guidance system of higher order than the simple first-order system considered up to now. A canonical fifth-order guidance system representation,[11] with transfer function

$$\frac{a_m(s)}{a_c(s)} = \frac{1}{[1 + (\tau/5)s]^5} \tag{15}$$

is taken to represent a third-order flight control system combined with a first-order guidance noise filter and a first-order seeker track loop. The form of Eq. (15) is selected so that when the denominator is expanded, the coefficient multiplying the first power of s (or the effective system time constant) is equal to τ. One possible drawback of this representation (which is usually chosen for convenience) is that a third-order autopilot exhibiting significant underdamped response as a result of a pair of complex poles in the transfer function cannot be accurately described by cascading three single-lag transfer functions.

Unlike the first-order guidance system, simple closed-form results for the miss of the fifth-order system in the unlimited acceleration case do not exist. Thus, the performance of the fifth-order system was evaluated numerically via Monte Carlo simulations, including the effects of acceleration saturation. The limits were again applied on the commanded missile acceleration, although in an actual implementation, some of the limiting might occur internal to the overall fifth-order transfer function model. This depends on the specifics of the particular design.

In Figs. 8 and 9, rms miss distance results are shown using the $M_{\rm rms}\omega^2/A_T$ normalization factor. As before, the validity of the factors for the saturated guidance system was verified over a range of target and missile parameters. The general trends indicate that the normalized miss distance increases as the acceleration limits are lowered until a particular value of $A_{\lim}/A_T$ is reached, and then the normalized miss reverses and begins to decrease. The value of $A_{\lim}/A_T$ at which this reversal occurs is about 2 for $N = 3$, 3 for $N = 4$, and 5 for $N = 5$.

Comparisons between the first- and fifth-order guidance systems are shown in Figs. 10–12 for values of $A_{\lim}/A_T$ of 3 and ∞. The

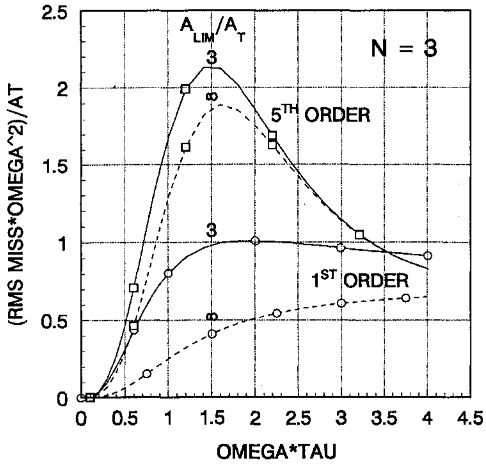

Fig. 10 Comparison of first- and fifth-order guidance system responses.

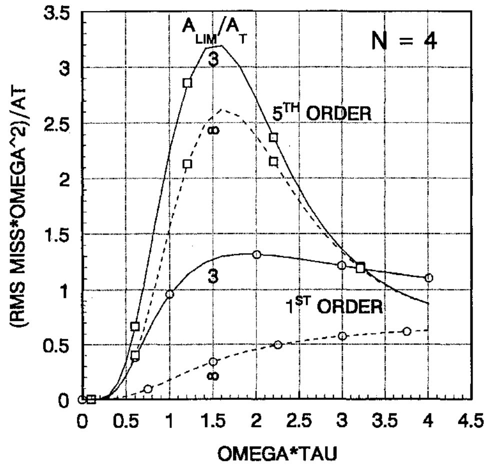

Fig. 11 Comparison of first- and fifth-order guidance system responses.

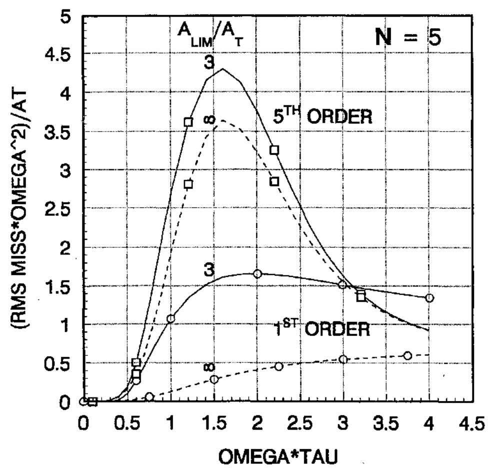

Fig. 12 Comparison of first- and fifth-order guidance system responses.

results indicate that there is a significant difference in the response of the two systems, with the higher-order system having a much larger normalized miss over the midrange of $\omega\tau$. Note also that the differences between unlimited and limited acceleration are less noticeable for the fifth-order system than for the first-order system.

Illustrative Example

It is useful to show some of these trends in terms of physical quantities with engineering units rather than normalized parameters. As an example, consider a scenario in which a tactical ballistic missile is undergoing weaving or spiraling motion as it re-enters the atmosphere after exoatmospheric flight. The results of Ref. 1 suggest that typical weave frequencies on the order of 0.5–1.5 Hz and accelerations in the range 1–8 g may be expected.

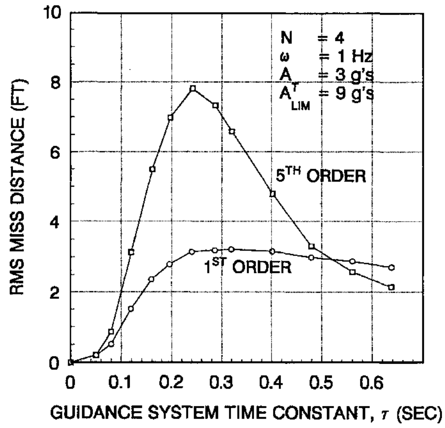

Fig. 13 Example rms miss vs missile time constant and system order.

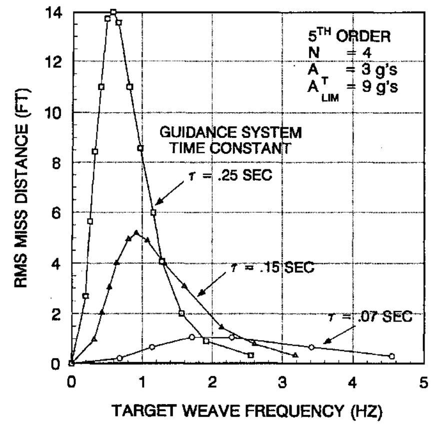

Fig. 14 Example rms miss vs weave frequency and missile time constant.

Figure 13 illustrates the rms miss distance of a PN homing missile against a 1-Hz weaving target with a maximum acceleration of 3 g. The interceptor has a navigation gain of 4 and is limited to a maximum acceleration of 9 g. The miss distance results are plotted as a function of the interceptor's time constant for first- and fifth-order guidance systems. Figure 13 clearly indicates that the idealized first-order system seriously underestimates the rms miss relative to the more realistic fifth-order system.

In Fig. 14, the variation of rms miss distance with target weave frequency is illustrated for the fifth-order system. Three values of missile time constant (0.07, 0.15, and 0.25 s) are shown. The results indicate that the overall weapon system requirements imposed on miss distance can place very stringent demands on the interceptor time constant. For example, if a fragmenting warhead with a sufficiently large lethal burst radius is used, then reasonably sized miss distances can be tolerated. On the other hand, if the missile is required to achieve hit-to-kill accuracies, then the demands on airframe responsiveness will be much tighter.

The example in Fig. 14 (which is not associated with any real weapon system) suggests the following. If 15-ft rms miss can be tolerated, then a time constant of 0.25 s is adequate. If 5-ft accuracy is required, then the time constant must be reduced to the level of 0.15 s. For a 1-ft rms miss associated with direct hit accuracies, the required time constant drops to a low value of 0.07 s. This example illustrates the severe demands that weaving target motion can place on the responsiveness of the interceptor. It also demonstrates the very close relationship that exists among performance requirements placed on the warhead, fuzing, and terminal guidance subsystems, and the need for a fully integrated and balanced system design.

Conclusions

The rms miss distance was proposed as an alternative means for evaluating interceptor performance against weaving targets. In contrast to peak miss, rms miss is a statistical measure taken over an ensemble of possible engagements, which reflects the random variations in target phase angle resulting from initial conditions at the start of terminal homing.

New closed-form results were derived for the ideal case of a single lag missile with unlimited lateral acceleration that provided insight into the behavior of miss distance with respect to target motion and missile guidance system parameters. The solutions revealed two possible normalization techniques for displaying rms miss distance data. These normalizations were shown to be valid representations when applied to the nonlinear case where acceleration is limited.

Significant performance differences between the first- and fifth-order guidance systems in the presence of saturation effects were demonstrated. An example showed the strong interaction among weave motion, acceleration levels, time constant, and miss distance. The example also illustrated the demanding requirements placed on missile time constant to achieve small misses against aggressively weaving targets.

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